

# Modeling the activities of firms under market conditions

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# Outline

1. Firm and production function
2. Perfect competition
3. Oligopoly
4. Conspiracy models and the cartel
5. Monopoly
6. Monopolistic competition

# Firm and production function



# Firm

- ▶ A firm is an administrative society of people united by certain interconnected activities.
- ▶ The purpose of the operation is to improve the living conditions and well-being of the people who work for this company.
- ▶ The purpose of rational management of the firm – to determine the quantity of products and necessary costs, if known prices for resources and products, the technical relationship between output and costs.

# The task of the firm

$$\begin{cases} TR \rightarrow \max \\ TC = \textit{const} \end{cases}$$

$$\begin{cases} TC \rightarrow \min \\ TR = \textit{const} \end{cases}$$

Maximize output at fixed costs

Minimize costs with a fixed output

# Formalization of the model of rational management

$$Z = \sum_{i=1}^n c_i x_i - \sum_{j=1}^m \sigma_j y_j \rightarrow \max$$

$$\sum_{i=1}^n a_{ij} x_i \leq b_j + y_j, \quad j = \overline{1, m},$$

$$x_i \geq d_i, \quad i = \overline{1, n}, \quad y_j \geq 0, \quad j = \overline{1, m},$$

where  $n$  - the number of products (services) produced by the firm

$m$  - the number of types of resources required

$c_i$  - unit price of  $i$ -th products

$x_{\text{and}}$  - production volume

$\sigma_j$  - unit cost of  $j$ -th resource

$y_j$  - the amount of resources required

$A_{ij}$  - technological matrix of resource costs per unit of output

$b_j$  - fixed assets

$d_i$  - forecast demand for the  $i$ -th products of the firm

# Production function

- ▶ The production function is the technological relation between output and costs.

$$q = f(\bar{x}) = f(x_1, \dots, x_m).$$

# Marginal rate of technological substitution

- ▶  $MRTS_{x_1x_2}$  indicates the amount by which one resource can be changed to another.

$$MRTS = -\frac{\partial x_2}{\partial x_1} = \frac{MP_{x_1}}{MP_{x_2}}$$



# Elasticity of substitution

$$\sigma_{lk}(x) = - \frac{d \ln \left( \frac{x_l}{x_k} \right)}{d \ln \left( \frac{MP_l(x)}{MP_k(x)} \right)}, l, k = 1, 2, \dots, n,$$

# Production function with constant substitution elasticity (CES)

$$q = l_0 \left( l_1 x_1^{-\beta} + l_2 x_2^{-\beta} \right)^{-h/\beta},$$

where

- ▶  $l_i > = 0$  ( $i = 0, 1, 2$ );
- ▶  $h$  is the degree of homogeneity,
- ▶  $\beta$ - substitution factor.

Elasticity: 
$$\sigma = \frac{1}{1 + \beta}$$

# Cost indicators

- ▶ General expenses (TC – total cost) – costs necessary for the production of the entire issue
- ▶ Variable costs (VC – variable cost) – costs that depend on the volume of production
- ▶ Fixed costs (FC – fixed cost) – costs borne by the firm regardless of output
- ▶ Marginal costs (MC – marginal cost) – additional costs required to produce an additional unit of output:

$$MC = \frac{\partial TC}{\partial q}$$

Signs \ types market	Perfect competition	Imperfect competition		
		Monopolistic competition	Oligopoly	Perfect monopoly
1. Number of firms	a lot	many	few	one
2. Types of products	standard	really or imaginary differentiated	a) standard, almost standard; b) moderately differentiated	Unique (no close substitutes)
3. Control	missing	certain	limited by interdependence	considerable, but where adjustable
4. Sales methods	market sale, auction	advertising and competitive qualities	advertising and competitive qualities	advertising and promotion
5. No price competition	is absent	very significant	very typical	advertising
6. Terms of market entry	very light	relatively light	there are obstacles	blocked
7. Availability of information	equal access	partially limited	some restrictions	certain restrictions
8. Spheres of economy	agricultural and a number of products	Retail	Steel, chemicals, cars	Utilities services, telephone,

# Perfect competition



# Perfect competition

- ▶ a large number of buyers and sellers;
- ▶ the same type of products;
- ▶ lack of opportunity for an individual seller and buyer to influence the equilibrium price;
- ▶ free entry into and exit from the industry;
- ▶ equal access to information (equal awareness of all market participants);
- ▶ lack of non-price competition.

If at least one condition is violated, then the market is my imperfect competition.

# Equilibrium condition for a perfectly competitive firm

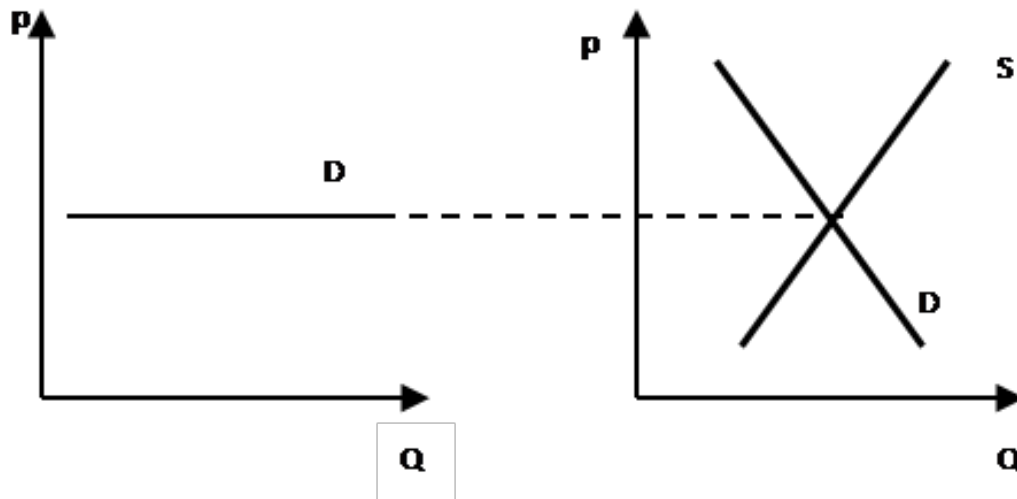
- ▶ In a competitive market, the demand line is both the average and marginal revenue line.

$$TR = pQ = R(Q)$$

$$AR = \frac{TR}{Q} = p,$$

$$MR = \frac{\partial TR}{\partial Q} = (pQ)' = p,$$

$$AR = MR = p$$



# Break-even condition

- ▶ In the short term, the firm may remain in the industry even in the presence of certain losses

$$p > \min AC$$



# Condition for closing the company

- ▶ in the short term can be obtained on the basis of complete loss

$$P(0) > P(Q) \text{ for } \forall Q > 0$$

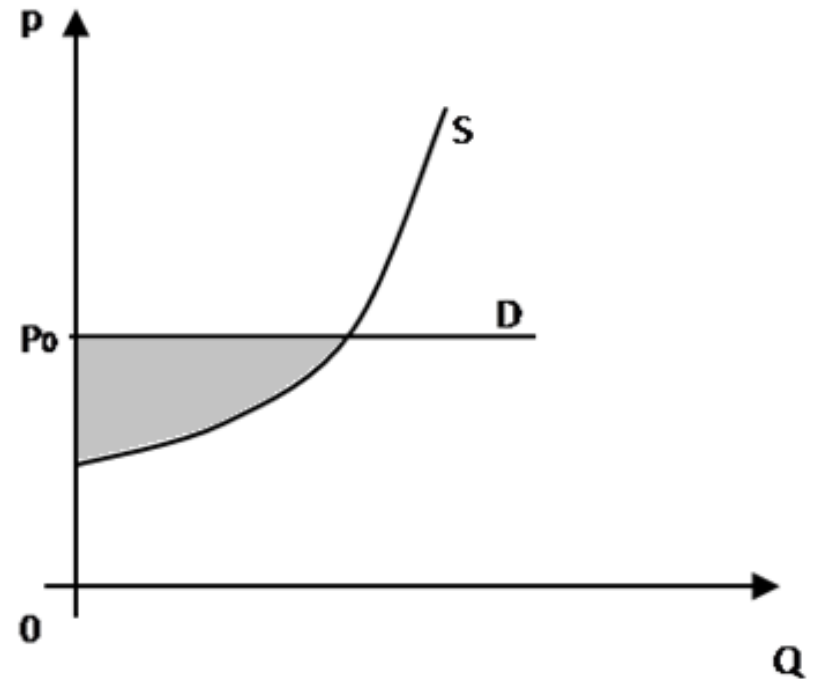
$$p < \min AVC(Q)$$

# Short-term competitive equilibrium

- ▶ **Short-term period** – period, during which the production capacity of each enterprise is fixed, but the output can be changed by changing the use of variables; while the total number of firms in the industry does not change.
- ▶  $P = MC$  – condition of effective output

# The supply curve of a competing firm

$$S = \begin{cases} MC, & p \geq \min AVC, \\ 0, & p < \min AVC \end{cases}$$



# Long-term competitive equilibrium

- ▶ Long-term period – the period during which production capacity can be adjusted to the conditions of demand and costs; the number of firms in the industry may vary
- ▶ In the long run, the firm cannot influence the market price, but can only choose the volume of output, thereby changing the AC, where

$$AC = \frac{TC}{Q}$$

# State of long-term equilibrium

- ▶ the firm has no motives to change the volume of output at its existing production assets;
- ▶ each firm is satisfied with the size of its existing production, as well as volumes;
- ▶ there are no motives that initiate the entry of new firms into the industry or the exit of old ones.
- ▶  $LMC = \min LAC$

# Long-term supply

- ▶  $\min LAC < p$  – companies are coming
- ▶  $\min LAC = p$  – equilibrium
- ▶  $\min LAC > p$  – the company is leaving the market

# Pareto efficiency principle

- ▶ An economic situation is Pareto-efficient if there is no way to improve the welfare of at least one economic agent without worsening the condition of any other economic agent.
- ▶ A situation of perfect competition is effective according to Pareto.

# Oligopoly





# Oligopoly

- ▶ An oligopoly is a market structure consisting of a small number of firms, several of which control a significant portion of the market and can influence price.
- ▶ Causes: economies of scale, barriers to market entry, mergers and acquisitions
- ▶ Signs of oligopoly:
  - in the market there are 2–5 firms and they control the whole market
  - standard or differentiated goods are sold
  - price control is limited by the interdependence of firms
  - barriers to entry into the industry
  - limited access to information
  - non–price competition is important, especially due to product differentiation.

# Models of oligopoly

	Simultaneous decision making	Consistent decision making
Number of products	Cournot model	Stackelberg model
Product price	Bertrand's model	Leadership in price

# Cournot model

- ▶ In the Cournot model, manufacturers simultaneously make decisions about product volumes based on their assumptions about the decisions of their competitors.

# Cournot model: description

$y_{ij}^e$  – expected by the manufacturer  $j$  the volume of production of the manufacturer  $i$ .

$y_{-j}^e$  – vector of expected volumes

$$(y_{i1}^e, \dots, y_{ij-1}^e, y_{ij+1}^e, \dots, y_{in}^e)$$

Expected income at volume  $y_j$  is equal to:

$$\Pi_j^e(y_j, y_{-j}^e) = p(y_j + \sum_{i \neq j} y_{ji}^e) \cdot y_j - c_j(y_j)$$

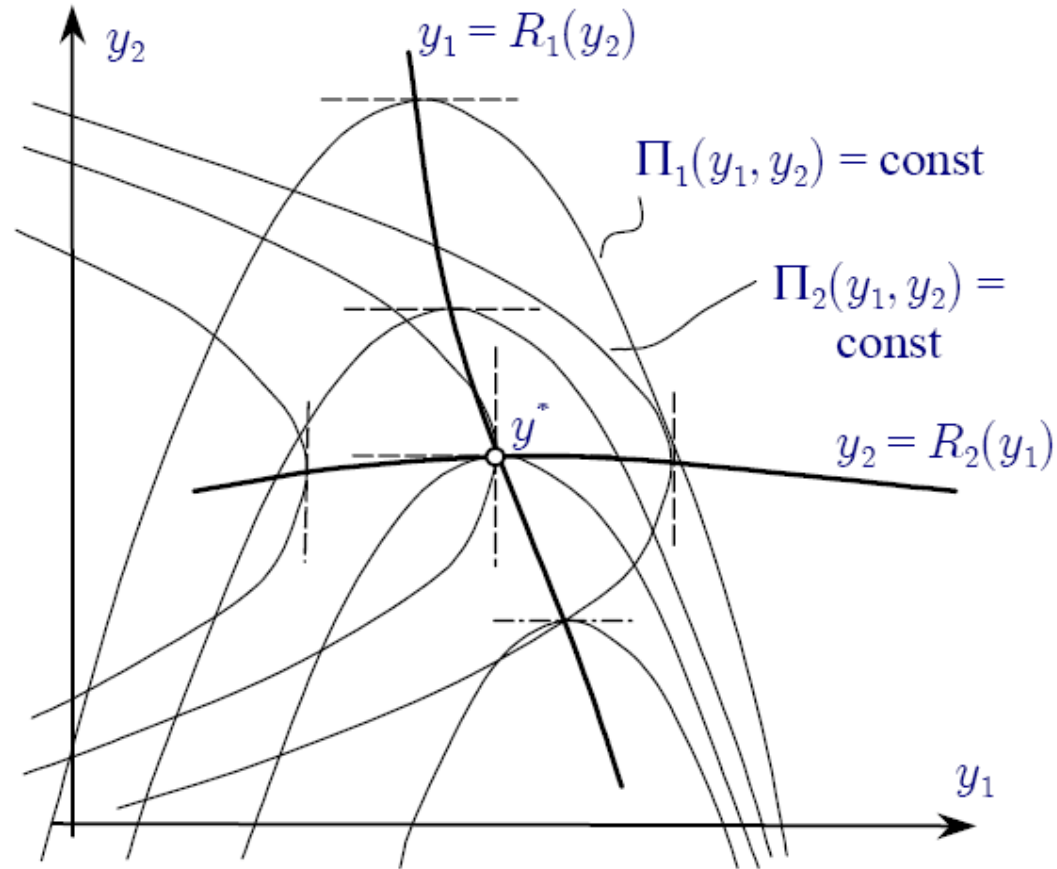
$R_j(Y_{-j})$  – reaction function of  $j$ -th manufacturer.

$Y_{-j} = \sum_{i \neq j} y_i$  – total expected volume of production by other participants

# Cournot equilibrium

$$\Pi_j^e(y_j) = p(y_i + \sum_{i \neq j} y_{ji}^*) \cdot y_i - c_j(y_i) \rightarrow \max_{y_i \geq 0}$$

# Graphic interpretation for two companies



# Cournot model's stability

- ▶ A study of the behavior of market participants in which the Cournot equilibrium is established shows that as the number of players in the market increases, the Cournot equilibrium approaches equilibrium in a competitive market, thereby increasing the welfare of society.

# Non-optimality of Cournot equilibrium

- ▶ In terms of oligopolists Cournot equilibrium is not optimal. Because if at least one of them reduce the volume of their products (by a fairly small amount), the profits of all oligopolists will grow.



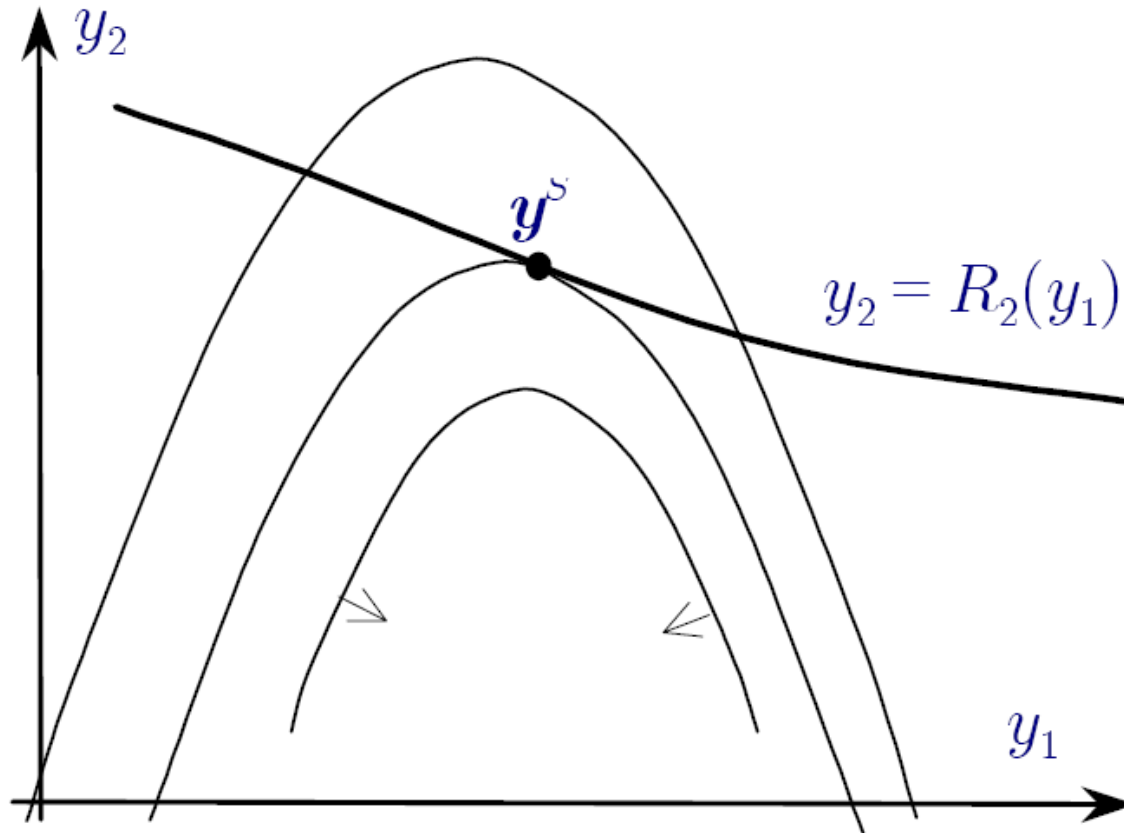
# Stackelberg model

- ▶ According to the model of Heinrich von Stackelberg, there are 2 players on the market. The first chooses the volume of output, he is the leader.
- ▶ The another player sets his output based on the already set volume of the leader's release, he is a follower.
- ▶ The leader calculates the actions of the follower, thereby maximizing his income.

# Stackelberg model: description

$$\Pi_1 = y_1 p(y_1 + R_2(y_1)) - c_1(y_1) \rightarrow \max_{y_1 \geq 0}$$

# Stackelberg equilibrium



# Bertrand's model

- ▶ oligopolists produce homogeneous products with fixed costs, the same for all manufacturers.
- ▶ manufacturers determine the best prices, which leads to oligopolistic price wars.

# Price war

- ▶ Price war – a cycle of consistent reduction in prices for products of firms operating on oligopolistic market. Profitable for consumers, but not profitable for manufacturers.
- ▶ The reduction may occur as long as the price is not equal to the average cost. The profit of firms in this case will be zero.

# Disadvantages of Bertrand model

- ▶ The model assumes that homogeneous products are produced, resulting in market tension.
- ▶ No restrictions on production volumes.
- ▶ Assumptions about the independence of marginal costs from output.
- ▶ The model is static.

# Elimination of shortcomings in Bertrand model

- ▶ Goods differentiation.
- ▶ Model with nonlinear increasing marginal costs.
- ▶ Dynamic modification of the Bertrand model.

# Goods differentiation – 1

- ▶ The consumer can distinguish goods by minimal differences and agrees to buy them at different prices.



# Goods differentiation – 2

Equilibrium in this model is achieved at a price that exceeds the marginal cost.

$$\left(1 - \frac{1}{\varepsilon_j}\right) p_j = c$$

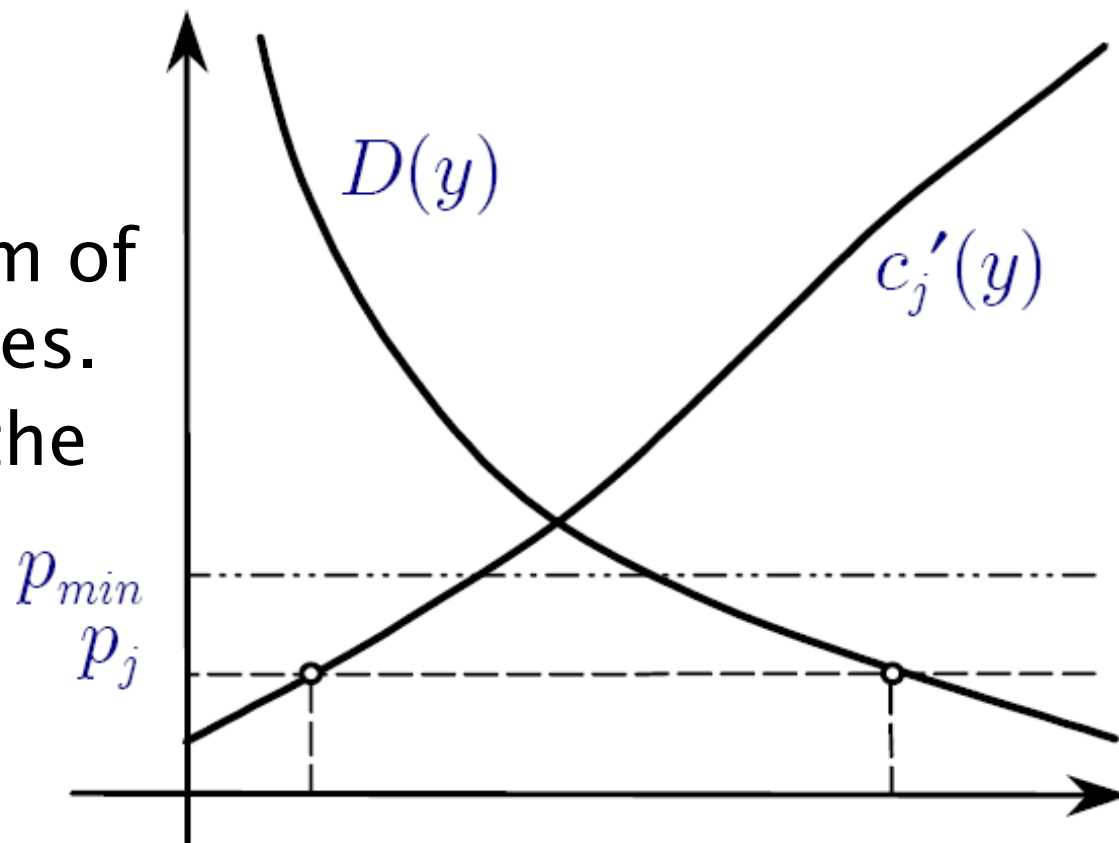
where  $\varepsilon_j$  is the elasticity of demand by price.

# Model with nonlinear increasing marginal costs – 1

- ▶ It may not be profitable for a firm to produce such a volume of products to satisfy the whole market at a price lower than that of its competitors.

# Model with nonlinear increasing marginal costs – 2

$P_{\min}$  – the minimum of competitors' prices.  
 $P_j$  – the price of the company j.



# Dynamic modification of the Bertrand model – 1

- ▶ The main difference from the classic model is that oligopolists take their price wars over an infinite number of time periods.
- ▶ Each firm maximizes its profits:

$$\Pi_j = \sum_{t=1}^{\infty} \delta^{t-1} \cdot \Pi_{jt}$$

$\Pi_j$  – firm profit in period  $t$ ,

$\delta$  – discount coefficient.

# Dynamic modification of the Bertrand model – 2

- ▶ The main task of the firm is to choose the price for the period  $t$ . It is believed that this price is a function of the "prehistory" of price competition.
- ▶ Balance in dynamic models Bertrand achieved only then, when discounting coefficient is less or equal  $1/n$ .

# Model of oligopolies with leader by price

- ▶ In this model, the leader firm sets the price of the product, all other firms choose the volume of output, given that the price of the product is already fixed.

# Model oligopolies with leader by price: description

- ▶ The firm leader leads like a monopolist in setting the price to maximize its profits.

$$\Pi_1 = D_1(p)p_1 - c_1(D_1(p)) \rightarrow \max_p$$

- ▶  $D_1(p)$  – the demand of the leader's firm, which is determined as follows:

$$D_1(p) = D(p) - \sum_{j=2}^n R_j(p)$$

# Model oligopolies with leader by price: functions reactions

- ▶ The reaction functions of other firms are the solution of the following equation:

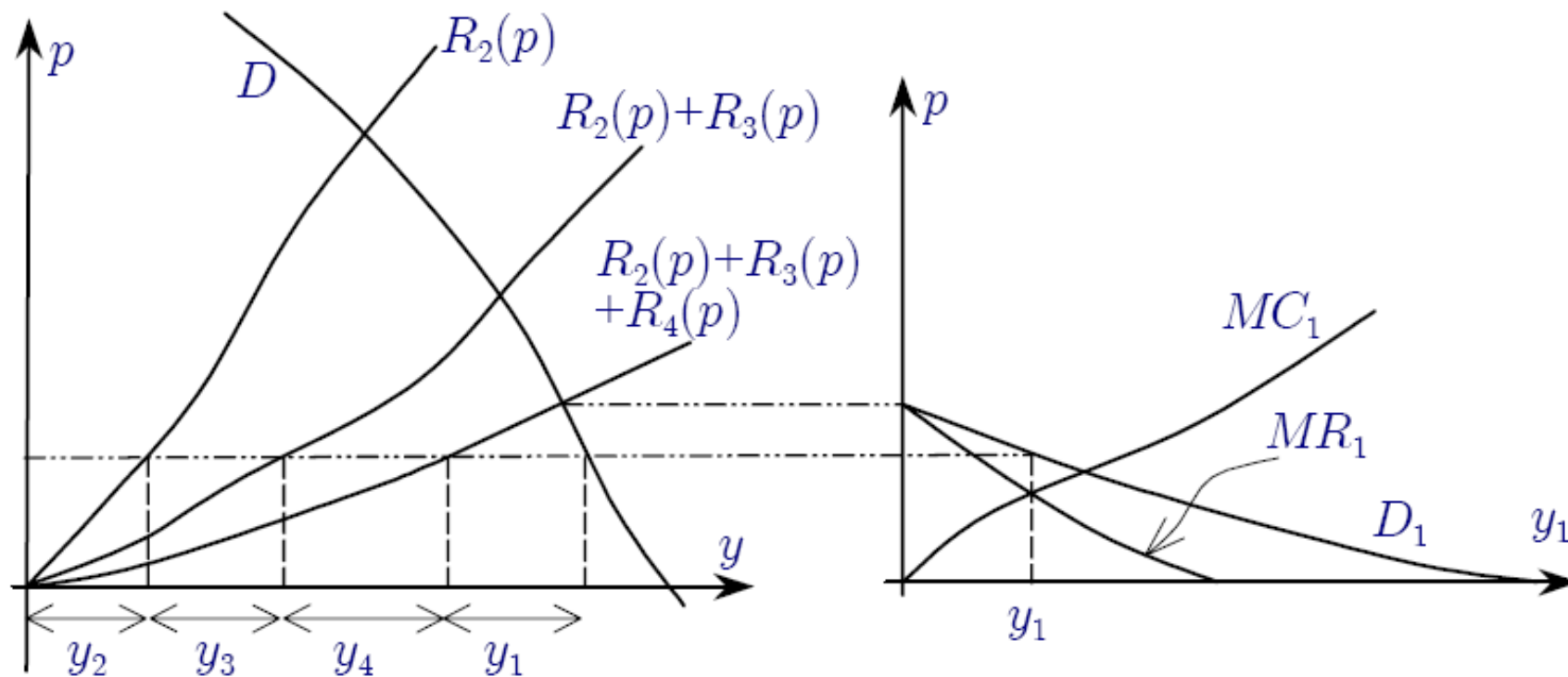
$$py_j - c_j(y_j) \rightarrow \max_{y_j \geq 0}$$

- ▶ It is also believed that the reaction functions are inverse to the marginal cost functions.

$$c'_j(R_j(p)) = p$$



# Model oligopolies with leader by price: balance with 4 players



# Conspiracy models and the cartel



# Conspiracy models and the cartel

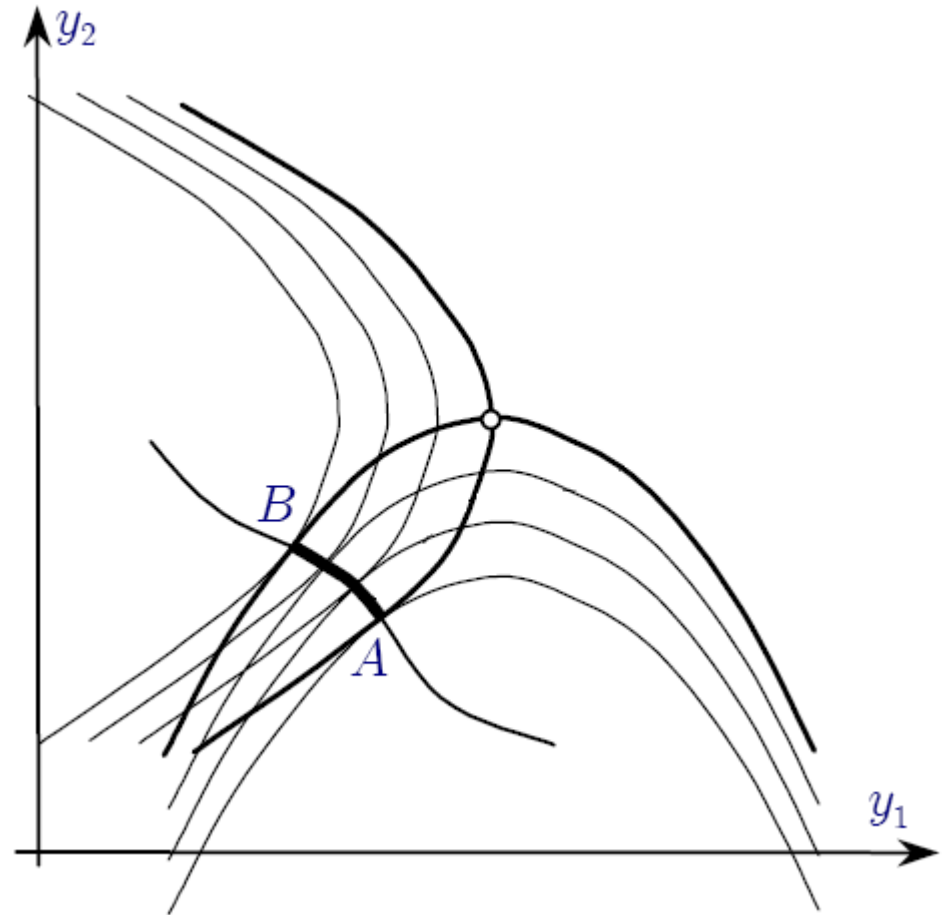
- ▶ An agreement with a possibility of redistribution of income between the oligopolists is advantageous for them and will be called a cartel.
- ▶ If there is no possibility of income redistribution, then there is a conspiracy

# Conspiracy model – 1

- ▶ Each of the participants at the conspiracy point will receive a profit not less than at the Cournot equilibrium point;
- ▶ the conspiracy point is effective (ie lies on the Pareto border) and there is no other point where at least one firm makes more profit.

# Conspiracy model – 2

As a rule, there can be many such points (segment AB) – a set of negotiations.



# The success of the cartel

- ▶ It is necessary to create a stable cartel organization, whose members agree on the prices and output levels of its members.
- ▶ Such an organization should have effective tools to penalize participants in the event of a cartel violation.
- ▶ The members of the cartel in the union must have the potential of a monopolist.

# Examples of cartels

- ▶ OPEC – International Agreement on Oil Producers;
- ▶ CIPEC – International Council of Copper Exporting Countries (Chile, Peru, Zambia, Zaire – together cover a third of world copper production)

# Cartel profit

$$\pi = P(q_1 + q_2)(q_1 + q_2) - c_1q_1 - c_2q_2 \rightarrow \max$$



# Monopoly



# Monopoly

- ▶ Monopoly is a type of market structure where only one firm offers the entire market volume of goods for which there are no close substitutes
- ▶ Conditions for the existence of monopolies:
  - all industry output is supplied by one firm, and the share of each consumer in the total is very small;
  - products are homogeneous and have no close substitutes;
  - entry of new firms into the market is blocked;
  - consumers do not interact with each other;
  - there is full awareness of market prices, volumes and customer demand.
- ▶ A market in which all 5 conditions are met is called a pure monopoly

# Measurement of monopolistic power – 1

- ▶ ***Concentration factor*** – is the percentage of total industry output, which provides a number of the largest firms in the industry. Concentration factors are most often used for 4 and 8 firms.

$$K = \frac{q_1 + q_2 + q_3 + q_4}{Q}$$

# Measurement of monopolistic power – 2

## ▶ *Herfindahl–Hirschman Index*

$$H = \sum_{i=1}^n \alpha_i^2,$$

$$\alpha_i = \frac{q_i}{\sum_{j=1}^n q_j}, i = \overline{1, n}$$

# Measurement of monopolistic power – 3

## ▶ *Lerner Index*

$$L = \frac{P - MC}{P}$$

Not always in the industry can be identified *MC*, then *MC* can be replaced by *AC*:

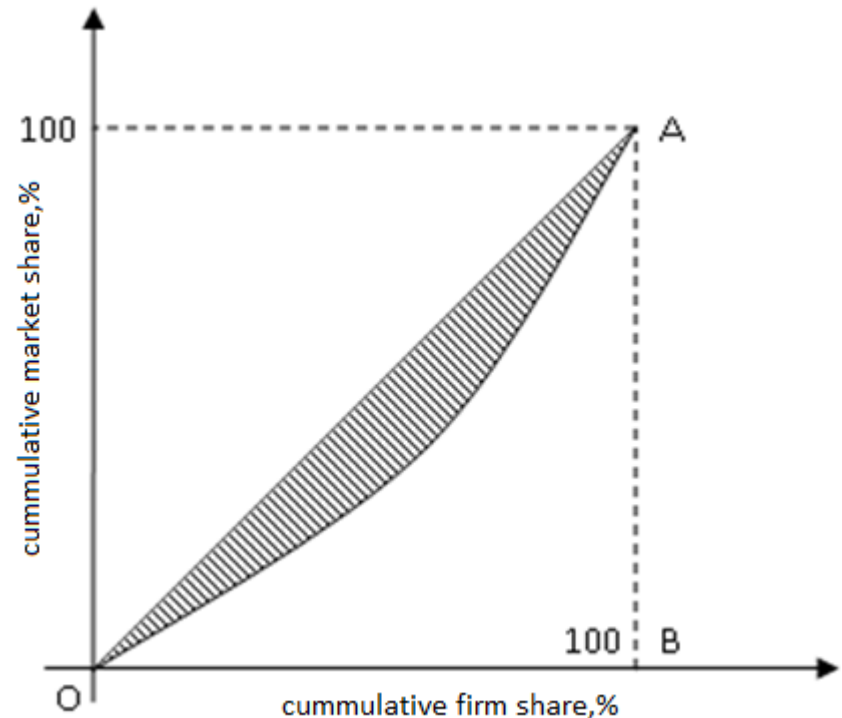
$$L = \frac{(P - AC)Q}{PQ} = \frac{\text{Profit}}{\text{Income}}$$

# Measurement of monopolistic power – 4

## ▶ *Ginny Coefficient* .

$$K = \frac{S_{OA}}{S_{OAB}}$$

If  $K = 0$ , it is a market of perfect competition;  
If  $K = 1$  – perfect monopoly.



# Measurement of monopolistic power – 5

- ▶ ***Entropy coefficient*** characterizes the degree of chaos or the disorder of the market in terms of the implementation of someone's individual will.

$$E = \sum_{i=1}^n Q_i \ln \frac{1}{Q_i}$$

$n$  – number of enterprises.

$Q_i$  – market share of the  $i$ -th enterprise in %.

The higher the value, the less sellers can influence the market situation.

# Monopoly equilibrium

Profit maximization condition for all types of market structures is described by the rule of marginal output:

$$\Pi = TR - TC \rightarrow \max$$

$$MR = MC$$

$$MR = p + Q \frac{dp}{dQ} = p \left( 1 + \frac{1}{E_D} \right)$$



# MR and elasticity

1.	Elastic demand	$\varepsilon_p < -1$	
		MR > 0	TR ↑
2.	Unity elasticity of demand	$\varepsilon_p = -1$	
		MR = 0	TR <sub>max</sub>
3.	Inelastic demand	$-1 < \varepsilon_p < 0$	
		MR < 0	TR ↓

# Monopoly equilibrium

- ▶ there is no supply curve;
- ▶ break-even point:

$$p^* = \min AC$$

- ▶ Closing point of the company:

$$p^* < \min AVC$$

- ▶ The best of the short-run equilibrium states is the long-run equilibrium state:

$$p > MR = MC = LMC$$

Short-term period

Long-term period

# Price discrimination

A situation in which this product is sold at more than one price to one or more customers.

## Conditions for price discrimination:

- ▶ the existence of a formal feature by which we can distinguish categories of consumers;
- ▶ the good cannot be resold.

# Price discrimination of the 1st degree (absolute)

- ▶ Occurs when the monopolist sets an individual price for each consumer at the level of his willingness to pay for the good, ie the maximum price that this consumer is willing to pay for this product.

# Price discrimination of the 2nd degree

- ▶ Set different prices depending on sales.
- ▶ Such price discrimination is possible with economies of scale

# Price discrimination of the 3rd degree

- ▶ It is used when it is possible to identify several groups of consumers with different elasticities of demand, ie to identify market segments. Market segmentation is carried out depending on certain characteristics that give the group the characteristics of consumption.

# Multi-period price discrimination

- ▶ It is close to price discrimination of the third type, the distribution of consumers in different categories depending on their demand functions is through the establishment of different prices in different periods.

# Pricing based on peak load

- ▶ Demand for some goods and services peaks in certain periods: for roads and tunnels – in peak hours, for electricity – summer evenings, for ski resorts and recreation areas – weekends.



# Two-stage payment

- ▶ Requires the consumer to pay for the right to purchase goods, and then the consumer pays extra for the consumption of each unit of goods. A classic example of this kind of pricing is the use of the telephone (people pay monthly subscription fee and every minute of conversation).

# Price discrimination

- ▶ **Advantages of price discrimination:**  
the monopolist, by changing the price in order to increase its own profits, gives the opportunity to buy goods to those buyers who could not sell them at face value.
- ▶ **Disadvantages of price discrimination:**
  1. increased price for buyers;
  2. dumping;
  3. protection of the market from competitors.

# Antitrust policy

- ▶ The position is considered an monopolistic, if market share of a particular product is 35% or less, and if the Antimonopoly Committee does not establish the presence of market power.

# Monopolistic competition



# Monopolistic competition

- ▶ Monopolistic competition is a type of market structure where:
  - there are many sellers and buyers in the market, the share of each of which in the volume of market sales is not significant;
  - products of different manufacturers are not homogeneous (differentiated);
  - entry into and exit from the market is free;
  - manufacturers do not interact with each other.
  - there is full awareness of market prices, volumes, consumer demand.

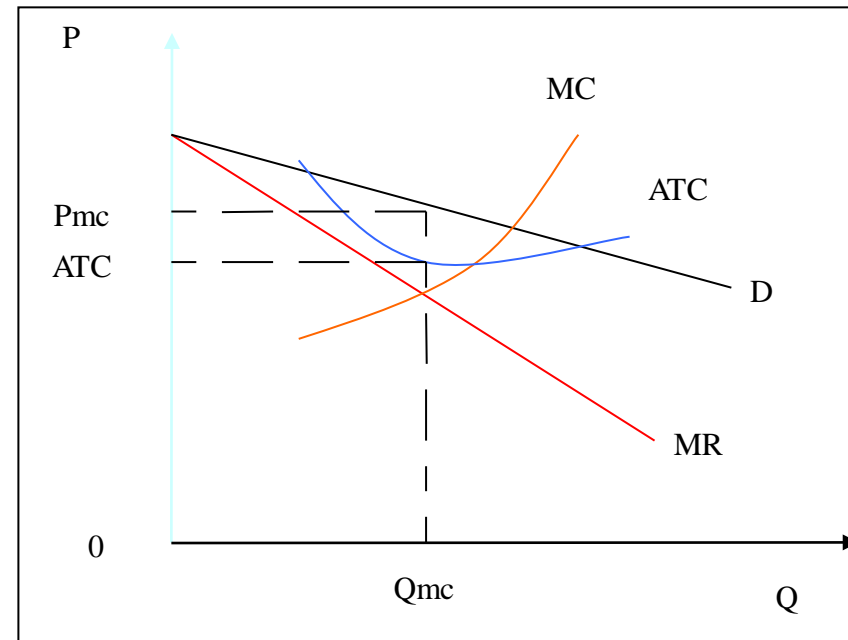
# Chamberlin model assumptions

- ▶ Due to differentiation, consumers are able to distinguish the products of different firms on the market, so the demand for products of a particular firm is not completely elastic, although it remains highly elastic. This means that firms have some market power and can vary prices (albeit very limited) without the risk of losing all buyers.
- ▶ The main feature of Chamberlin model there is an absolute symmetry of the position of all firms in this industry.

# Chamberlin's model – 1

- ▶ In the short term, the firm can maximize profits or minimize losses, following the general rule

$$MR = MC$$



The short-run equilibrium of a monopolistic competition is similar to the equilibrium of a perfect monopoly

# Chamberlin's model – 2

- ▶ In the long run, in the case of profitability in the industry begin to enter new firms, attracted by the opportunity to make an economic profit.
- ▶ In a state of long-term equilibrium, all firms in the industry receive only normal profits.



# Thank you!